Thus the solution satisfying the boundary conditions is given by

$$g' = \varphi = \tfrac{x}{3}(17 - 20x^3) + \tfrac{5}{9}(1 - x^3)(1 - 4x^3) \times \{\ln[(1 - x^3)/(1 - x)^3] + 2\alpha \arctan[(1 + 2x)/\alpha] - \pi/\alpha\} \quad (7)$$

It is noted that

$$g''(0) = (d\varphi/d\eta)_{\eta=0} = \tfrac{1}{3}(d\varphi/dx)_{x=0} = 3 \quad (8)$$

This agrees with Plotkin's numerical result.

References

[1] Plotkin, A., "A Second-Order Correction to the Glauert Wall Jet Solution," *AIAA Journal*, Vol. 8, No. 1, Jan. 1970, pp. 188–189.

[2] Glauert, M. B., "The Wall Jet," *Journal of Fluid Mechanics*, Vol. 1, Pt. 6, 1956, pp. 625–643.

Reply by Author to N. Hayasi

Allen Plotkin

University of Maryland, College Park, Md.

The author would like to thank Dr. Hayasi for providing an analytical solution to Eq. (1). He would also like to acknowledge the receipt of an equivalent analytical solution from Dr. Stephen Maslen in a private communication.

Received May 4, 1970.

* Assistant Professor, Department of Aerospace Engineering.

Comment on "Asymptotic Description of Radiating Flow near Stagnation Point"

Walter B. Olstad*

NASA Langley Research Center, Hampton, Va.

In a recent article, Jischke[1] applied the method of matched asymptotic expansions (MAE) to obtain solutions for the stagnation-region flow of an inviscid, gray radiating shock layer when either the radiation convection energy parameter Γ or the reference optical depth τ_s is small. He also states that the Poincare-Lighthill-Kuo (P-L-K) technique used earlier by the present author to obtain solutions to the same problem[2–4] (formulated in a more general way to include nongray radiation and to preserve the coupling between the energy and momentum equations) fails to correctly resolve the singular behavior. It is the purpose of this comment to explore the nature of this "failure" and to show that it is a failure only in a rather restricted sense and in no way invalidates the results obtained using the P-L-K technique.

As Jischke points out the P-L-K solution fails to correctly resolve the singular behavior at the stagnation point ($x = 0$) because $h_{P-L-K}(0) \neq h_{exact}(0) + 0[\delta(\epsilon)]$, where h represents the enthalpy of the gas and $0[\delta(\epsilon)]$ represents the order of the approximation elsewhere in the interval (0, 1). However, there is ample justification for ignoring this difficulty of the P-L-K technique as it is recognized that the value of the enthalpy of the gas adjacent to the wall obtained by any inviscid analysis is physically incorrect and hence of little practical interest. The primary value of the inviscid analysis is in displaying the behavior outside the viscous and thermal boundary layers. If this viscous boundary layer is smaller than $0(e^{-1/\Gamma})$ then one must carry the inviscid solution to higher order using either the MAE or P-L-K techniques to provide a common overlap region in which the inviscid and viscous solutions are both valid. The inviscid analysis is also a valuable tool (again for a thin boundary layer) for obtaining global quantities such as the shock detachment distance and the radiative heat flux. It should be noted that the global quantities (particularly the nongray radiative heat flux) obtained with the regular perturbation (or outer) solution are not sufficiently accurate for engineering purposes. The P-L-K technique was adopted by the present author to improve on the accuracy of these global quantities. In this regard the first-order MAE solutions of Jischke yield global quantities evaluated with the first-order regular perturbation expansion which provides no improvement. Of course, an improvement could be obtained by using his composite solution.

There is also another reason for not being too concerned over the failure of the P-L-K technique to provide a uniformly valid solution. An approximate solution is considered to be uniformly valid if it satisfies the condition $|y_{exact}(x) - y_{approx}(x)| < \delta(\epsilon)$, for all x, in the interval of interest, where $\lim \delta(\epsilon) \rightarrow 0$. Since the P-L-K technique requires expansion of the independent variable as well as the dependent variable one might expect that the aforementioned condition is too strict and that satisfaction of a somewhat more relaxed condition should be sufficient. Such a condition might be $|y_{exact}(x) - y_{P-L-K}[x + \lambda(\epsilon)]| < \delta(\epsilon)$ for all x in the interval of interest. Here $0(\lambda) \ll 0(\delta)$. This condition is satisfied by the P-L-K solutions to the inviscid, radiating, stagnation-region flow problem for either Γ or τ_s small. It is also satisfied by the P-L-K solution of Jischke's example problem, $xy' = \Gamma(y^2 - \alpha^2)$, $y(1) = 1$, for either Γ or α small.

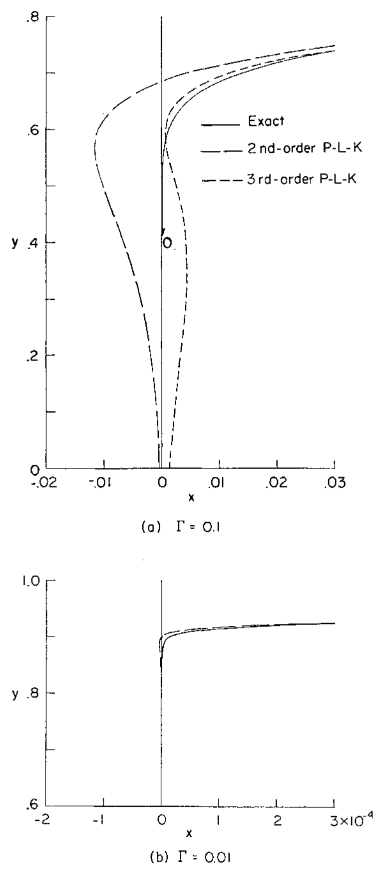

Fig. 1 Comparison of P-L-K and exact solutions for Γ small.

Received March 27, 1970.

* Head, Gas Physics Section, Aero-Physics Division. Member AIAA.